

# 1.7

## Focus on Reflecting

When you solve a problem, you need to regularly check and re-check your thought processes.

- Is your strategy the best one?
- Are you checking to make sure you are not making any errors?
- Have you considered alternative strategies or the use of different tools?
- Does your answer make sense?
- Can you verify that your solution is correct?

Reflecting on your processes will make you a better problem solver.



### Tools

- paper strips
- tape
- pencil

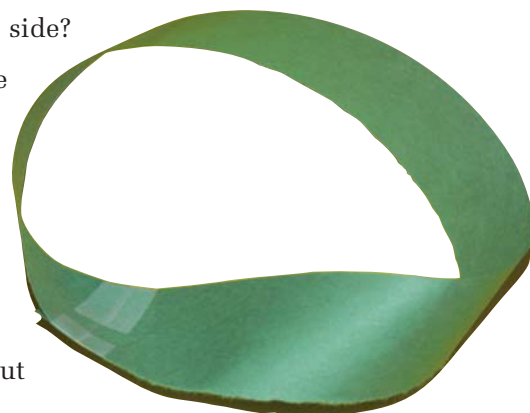
## Investigate

### How can the process of reflecting help you solve problems?

#### A: Möbius strip

How can a sheet of paper have only one side?

1. Cut a strip of paper about 4 cm wide and 28 cm long. Twist it a half turn and tape the ends together. This is called a Möbius strip.
2. Draw a line along one face of the paper without lifting your pencil. Describe what happened.
3. Predict what would happen if you cut all the way along this line.
4. Test your hypothesis. What happened? Explain.
5. Make another Möbius strip. This time, draw two lines, each one-third of the distance from each edge. Then, cut along the two lines.
6. **Reflect** What happened? Can you explain why the result was different?



#### B: Strategies

How many numbers between 1 and 100 are divisible by 7?

1. Devise two strategies that would be efficient in solving this problem.
2. Solve the problem in two ways, using both strategies.
3. **Reflect** Explain which strategy was more effective.

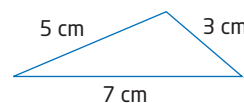
## Key Concepts

- As you solve a problem, indicate, in words or symbols, what your strategy is, so you can evaluate it later on.
- Reflect back to evaluate the effectiveness of your strategies and to verify that your solution is correct.
- If you determine that the solution was incorrect, check to see if your steps were done correctly, or try a different strategy.



## Communicate Your Understanding

- C1** To answer the problem, “Find five odd numbers that add up to 55,” Ben used the strategy of systematic trial.
- Ben’s first trial was  $3 + 5 + 7 + 9 + 11 = 35$ . What might his next trial be? Why?
  - Do you think that systematic trial is a good strategy for this problem? Explain.
- C2** A triangle has sides of length 3 cm, 5 cm, and 7 cm. Describe how you can verify whether this is a right triangle.
- C3** Using the drive-through one night at a local coffee shop, Tara ordered a large coffee and a muffin. The drive-through attendant told her that her total was \$19.38. Tara argued with the attendant that it is not possible for the total to be \$19.38 when she only ordered a coffee and a muffin. The attendant insisted the total was \$19.38.
- Explain what the drive-through attendant should have done when Tara questioned the total.
  - Discuss what the drive-through attendant might have done wrong.



## Practise

For help with question 1, see the Investigate, Part A.

- To create a double Möbius strip, place two strips of paper together, one on top of the other. Twist them a half-turn and tape them together, end to end, at both the top and bottom. Then, cut the strip down the middle. Describe the results and explain why it happened.
- If you multiply a number by  $-7$ , and then add 12, the result is  $-380$ . What is the number?
- Half of a number, decreased by  $\frac{3}{4}$ , gives  $\frac{7}{12}$ . What is the number?
- What strategies did you use in questions 2 and 3? How effective were they?

5. What is the least number divisible by all of the integers 1 through 9? Explain your strategy and whether it was effective.

### Connect and Apply

6. a) Using only the integers  $-3, -2, -1, 0, 1, 2$ , and  $3$ , find all solutions to  $X + Y + Z = -5$ . You may repeat numbers.  
 b) What strategy did you use? Was it effective? Explain.
7. a) Multiply  $1 \times 2 \times 3 \times 4$ .      b) Calculate  $5^2 - 1$ .  
 c) Multiply  $2 \times 3 \times 4 \times 5$ .      d) Calculate  $11^2 - 1$ .  
 e) Multiply  $4 \times 5 \times 6 \times 7$ .      f) Calculate  $29^2 - 1$ .  
 g) Describe a rule that this pattern seems to illustrate.  
 h) Verify your rule by trying it with two more examples.  
 i) Did your examples work? If not, try to develop a different rule and verify it.

### Literacy Connections

A pixel is a single grid point on a computer or TV screen. Many screens are 640 pixels across and 480 pixels high.

8. In a video game, a character has been programmed to start at 60 pixels to the left of centre. The character moves 90 pixels to the right, then 75 pixels to the left, then 60 pixels to the right, and so on.  
 a) The character disappears when it lands on zero, the centre. After how many moves will this occur?  
 b) Verify that your answer is correct.
9. How many numbers between 1 and 100 are divisible by either 2 or 3? Explain your strategy and verify that it works.
10. A recipe calls for 1 kg of flour, 500 mL of milk, 2 eggs, and 125 g of sugar. To triple the recipe, Karen calculated that she needed 3 kg of flour, 15 L of milk, 6 eggs, and 3.75 kg of sugar.  
 a) Without calculating the correct amounts, find Karen's errors, and describe the clues to her errors.  
 b) Verify that your discoveries are correct.

### Extend

11. How many square metres of pizza are ordered in Ontario in a year?
12. In a magic square, the rows, columns, and diagonals each add to the same sum.  
 a) Construct a magic square using these numbers:  
   i)  $1, 2, 3, 4, 5, 6, 7, 8, 9$   
   ii)  $-4, -3, -2, -1, 0, 1, 2, 3, 4$   
 b) Describe the strategies that you used.

